Multivariable CALCULUS WORKSHEET: Complete All Problems

PRE-CALCULUS:

1. Sketch the graph and determine its **DOMAIN**:

   a: $\dfrac{(x+3)(x+2)}{(x-4)(x+1)}$ **b:** $f(x)=\dfrac{x^2-25}{x+5}$ **c:** $f(x)=\dfrac{(x^2-4)(x-3)}{x^2-x-6}$

   Include the DOMAIN and RANGE also for the rest:

   d: $f(x)=\begin{cases}9-x^2 & x\neq -3\\ 10 & x=-3\end{cases}$ **e:** $f(x)=\lfloor x\rfloor$ ($\lfloor x\rfloor$ means the greatest integer less than or equal to x)

   f: $f(x)=3\lfloor 2x\rfloor$ **g:** $f(x)=\begin{cases}x+3\,, x<-5\\ \sqrt{25-x^2}\,, -5\le x\le 5\\ 3-x\,,\ x>5\end{cases}$ **h.** $f(x)=\lfloor x-4\rfloor$

2. The **unit step function** is defined to be $U(x)=\begin{cases}0, & x<0\\ 1, & x\ge 0\end{cases}$. Sketch the graph of

   a: U(x) b: $xU(x)$ c: $(x+1)U(x+1)-xU(x)$

3. The **signum function** is defined to be : $\operatorname{sgn}(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$. For each function below, sketch the graph. a: x sgn(x) b: $x-2\operatorname{sgn}(x)$

4. Write this function as a piecewise function (like #1d or #1g) **without** using the absolute value symbol: $|x^2-1|$. Sketch a graph of this function.

5. If $f(x)=\dfrac{x+1}{x-1}$ *and* $g(x)=\dfrac{1}{x}$, write a **formula** for and give the **domain** of:

   a: $f\cdot g$ b: $f\circ g$ c: $f\circ f$

6. If $f(x)=\sqrt{x-2}$ *and* $g(x)=x^2-2$, write a **formula** for and give the **domain** of:

   a: $f\circ g$ b: $g\circ f$ c: $f\circ f$

7. Express $h(x)=\sqrt{x^2-4}$ as the composition of two functions, f and g $(f\circ g)(x)$, in **THREE** different ways.

8. Is $f(x)=\sqrt[3]{x}$ even, odd or neither? Prove analytically (algebraically) using the definition (**NOT** graphically/geometrically). Look up the definition if necessary.

9. Is $f(x)=\dfrac{x^2-5}{2x^3+x}$ even, odd or neither? Prove analytically (**NOT** graphically).

10. Write the function $f(x)=|x-2|-|x+2|$ without using absolute value bars AND state whether f(x) is even, odd or neither with a defense for your answer.

11. If f and g are two functions such that when composed in either order, the result is the identity function then f and g are inverses of each other. If $f(x)=x^2,\ x\le 0$ *and* $g(x)=-\sqrt{x}$, show that they are inverses analytically via the above definition. Be SURE the details are clear!

12. If $f(x)=x^2$, find TWO functions, g, for which $(f\circ g)(x)=4x^2-12x+9$.

13. If; ***f(x)*** = {(3,5), (2,4), (1,7)}, ***g(x)*** = $\sqrt{x-3}$, ***h(x)***= {(3,2), (4,3), (1,6)}, ***k(x)*** = x^2+5, determine each of the following:

a: $(f+h)(1)=$ b: $(k-g)(5)=$ c: $(f\circ h)(3)=$ d: $(g\circ k)(7)=$

e: $f^{-1}(x)=$ f: $g^{-1}(x)=$ g: $\dfrac{1}{f(x)}=$

14. Write the inequality $|A|<B$ without absolute value bars.

15. Write the inequality $|A|>B$ without absolute value bars.

16. Solve for x: $|2x-3|<5$

17. Solve for x: $|3x-2|>5$

18. Do you think it is true that $2^{2n+1}+1$ is divisible by 3 for all $n\ge 1$. If you believe it might be, prove it by induction.

Solve for x showing all work:

19. $(x-2)(x+3)^7(x-14)^{18}(x+11)^{29}(x)^{34}>0$ 20. $27^{2x}=9^{x-3}$

21. log x + log(x – 3) = 1 22. $e^{3x}=5$ 23. ln y = 2x - 3

State the following formulae: (from memory if at all possible, only look up what you MUST). **Be sure you know these by the first day of class! They will be assumed**

24. $\sin(A+B)=$

25. $\cos(A+B)=$

26. $\sin 2A=$ **Show the derivation** of this from #24 and/or #25, be CLEAR

27. $\cos 2A=$ = = (3 forms) **Show the derivations** from #24 and/or #25

28. $\sin(\frac{1}{2}A)$ **Show the derivation** from #27, be CLEAR

29. $\cos(\frac{1}{2}A) =$ **Show the derivation** from #27, be CLEAR

30. $\sec^2 A =$ **Show the derivation** from $\sin^2\theta + \cos^2\theta = 1$, be CLEAR

31. $\csc^2 A =$ **Show the derivation** from $\sin^2\theta + \cos^2\theta = 1$, be CLEAR

32. In what quadrant is the terminal side of a 100 radian angle that is in standard position? Explain.

Suppose that the trigonometric functions were defined in exactly the way you have learned based on the coordinates of a point where the terminal side of the angle meets the unit circle **BUT** instead were based on a **UNIT SQUARE** instead of a unit circle. A unit square is ONE unit on each side, centered on the origin with sides parallel to the axes. With this **one** modification, what would the values of these trig functions now be? **(NO Calculator)**[**CORRECT DEFINITIONS BASED ON UNIT CIRCLE**: The sin(Theta) = the y coordinate of the point on the unit circle where the terminal side of Theta intersects the circle. The cos(Theta) is the x coordininate of the point on the unit circle where the terminal side of Theta intersects the circle]

33. $\sin\frac{\pi}{4} =$ 34. $\tan\frac{\pi}{4} =$ 35. $\cos\frac{3\pi}{4} =$ 36. $\csc(-\frac{19\pi}{4})$ 37. $\sin 30^o =$

38. $\cos(120^o) =$ 39. $\tan(-210^o) =$ 40. $\csc(1260^o) =$ 41. $\cos(43.72198^0) =$

42. $\sin(-948.6671^0) =$ 43. $sin^2315^\circ + cos^2315^\circ =$

44. State a valid theorem from Geometry who converse is NOT valid.

45. State a valid theorem from Geometry who converse IS valid.

46. State a valid theorem from Algebra who converse is NOT valid.

45. State a valid theorem from Algebra who converse IS valid.

46. State a valid theorem and its contrapositive.

INTEGRATION:

Perform the following integrations, #1-#18, by **'thinking reverse differentiation'**. **DO NOT use any methods (like substitutions)**. You may do a little algebraic simplification but NO calculator. Not sure about these, email and ask please as you may not have seen these previously. These are indefinite integrals. Email me if a sample is needed on any of these:

1. $\int 11x^{\frac{7}{5}}dx$ 2. $\int \frac{7}{\sqrt{y}}dy$ 3. $\int w^7(2w^5 - 3w^6)dw$

4. $\int \sqrt[3]{y}(y^4+1)dy$

5. $\int \frac{x^4+2x^2-1}{\sqrt[3]{x}}dx$

6. $\int 5\cos(2y)-4\sin(7y)dy$

7. $\int \frac{\cos x}{\sin^2 x}dx$

8. $\int \frac{3\tan\theta-4\cos^2\theta}{\cos\theta}d\theta$

9. $\int x\left(\sqrt[3]{x^2-9}\right)dx$

10. $\int \frac{y^3}{(1-2y^4)^5}dy$

11. $\int \frac{w^3+w^2+w+1}{w+1}dw$

12. $\int 17x^2\sin(2x^3)\,dx$

13. $\int \sqrt{1+\frac{1}{3x}}\frac{dx}{2x^2}$

14. $\int \frac{\sec^2(3\sqrt{t})}{\sqrt{t}}dt$

15. $\int \frac{x^2+2x}{\sqrt{x^3+3x^2+1}}dx$

16. $\int w(w^2+1)\sqrt{4-2w^2-w^4}dw$

17. $\int \sin y\left[\sin(\cos y)\right]dy$

18. $\int(\tan 2x+\cot 2x)^2 dx$

Integrate by parts (check your answer by differentiating it), show all work clearly including the substitutions being used: Look this up if unfamiliar.

19. $\int t\cos t\ dt$

20. $\int x^2e^x dx$

21 $\int \frac{x^3dx}{\sqrt{1-x^2}}$

22. $\int \cos(\sqrt{x})\,dx$

Integrate using a Trig Substitution: Look this up if unfamiliar.

23. $\int \frac{\sqrt{9-x^2}}{x^2}dx$

Definite Integrals (Evaluate):

24. $\int_0^{3\pi} \sin^2 5x\,dx$

25. $\int_0^{\pi} \sin^3 2x\,dx$

26. $\int_0^{\frac{\pi}{4}}(1+e^{\tan\theta})\ \sec^2\theta\,d\theta$

For #25-26 use any methods at your disposal as long as you make it clear how you are proceeding. Explain your thinking as clearly as possible. ANSWER TO FOUR DECIMAL PLACES. **DO NOT** use trial and error. You may use a calculator. Your explanation may include a graph sketch if it helps your clarity.

27. Consider the function f(x)=3x-4. It is clear that f(5)=11. What is the largest (longest) **open interval** containing 5 in order that all the 'x' values in the interval have their corresponding 'y' values less than 0.06 units from 11? Show calculations.

28. Consider the function $f(x) = x^2 + 4$. What is the largest (longest) **symmetric** (ie 2 is the center) open interval containing 2 in order that all the 'x' values in the interval have f(x) values less than 1 unit from 8? Show calculations to the nearest hundredth.

Given useful formulas:

A: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

B: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

C: $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

D: $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$

29. Check formula B above for three different values of 'n' where n>3. Show all details.

30. Check formula C above for two different values of 'n' where n>2. Show all details.

31. Calculate $\int_0^4 x^2 dx$ by writing and then evaluating the Riemann Sum that this integral represents. You may use summation notation or just write out the terms explicitly. Check your final answer by evaluating this integral using the Fundamental Theorem of Calculus. The above formulas are available for use here.

32. Calculate $\int_1^3 2x^3 + 5x\, dx$ by writing and then evaluating the Riemann Sum that this integral represents. You may use summation notation or just write out the terms explicitly. Check your final answer by evaluating this integral using the Fundamental Theorem of Calculus. The above formulas are available for use here.

33. Calculate $\int_2^6 5x^2 + 7x + 3\, dx$ by writing and then evaluating the Riemann Sum that this integral represents. You may use summation notation or just write out the terms explicitly. Check your final answer by evaluating this integral using the Fundamental Theorem of Calculus. The above formulas are available for use here.