AP Physics Summer Assignment
Due: the first day of school

Present your work on separate paper, not along the margins of this handout.
Save your work as a single pdf file, which you will submit on Canvas on the first day of school.

Preliminaries

Pendulums are everywhere: in grandfather clocks, on musical metronomes, and on playgrounds (swings).
A pendulum is a mechanical system: a massive bob, attached to a string or rod, that swings back & forth in periodic motion.
To simplify the mathematics, we'll assume that the rod is massless and consider only its length.

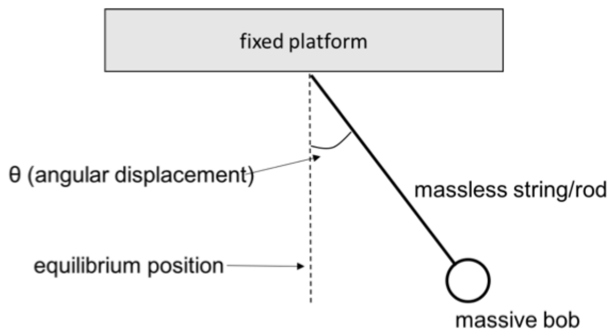


Figure 1: diagram of a basic pendulum

Build Your Pendulum

1.
Build your own pendulum with a piece of string, a platform or hook to hang the string, and something with mass for the bob.
Keep handy the following:
extra string & scissors
metric ruler (to measure lengths)
second bob (either heavier or lighter, but not the same weight)
protractor (to measure angles)
calculator & timer

2.
Once your pendulum is built, measure the length from the pivot point to the center of the bob.
We will call that length L.
Use meters and be accurate to three decimals. So, with a centimeter ruler, be accurate to one decimal.

3.
Displace the pendulum 15 degrees from equilibrium (vertical), and then release it to swing back & forth.
Record the time needed to complete 10 oscillations. Note: one oscillation equals one complete back & forth motion.
Divide that elapsed time by 10, to arrive at the time needed to complete one oscillation.
That time interval that you just measured is called the pendulum's "period".

What Affects a Pendulum's Period?

4.
Does the initial angular displacement affect your pendulum's period?
To answer this question, repeat step 3 twice with two other initial displacements.
I suggest using 5 degrees and then 10 degrees.
Don't exceed 15 degrees; doing so introduces other mathematical complications.
Record your result and, in a sentence or two, explain what you learned.

5.
Does the bob's mass affect your pendulum's period?
To answer this questions, replace the original bob with the second bob, either heavier or lighter.
Repeat step 3.
Record your results and, in a sentence or two, explain what you learned.

6.
Does the string's length affect your pendulum's period?
To answer this question, repeat step 3 twice with two different string lengths.
I suggest using lengths $\frac{1}{2}L$ and then $2L$, where L represents your string's original length.
Record your results and, in a sentence or two, explain what you learned.

Conduct an Experiment

What you've discovered so far is quite surprising:
A pendulum's length affects its period, but its mass and initial angular displacement do not! How amazing is that?
So, if we wish to use a pendulum to mark time, any mass and small displacement will do. We just have to find the correct length.

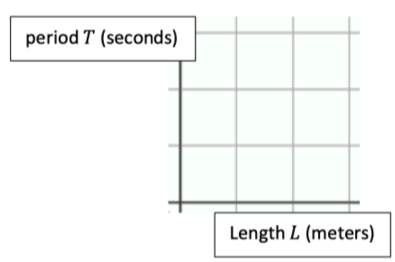
Given that insight, let's conduct an experiment to explore how length L controls the period T.
Length L will be the independent variable.
Period T will be the dependent variable.

7.
You've already calculated the periods for three different lengths.
On your paper, copy the table shown, and record those results in the table's first three rows.

length L (meters)	**period T (seconds)**

8.
Now repeat step 3 for three more lengths.
The broader the range of lengths you can use, the better.
Record those results in your table's remaining rows.
When done, you should have six lengths and their corresponding periods.

9.
Plot your results:
The horizontal axis should represent lengths (in meters).
The vertical axis should represent periods (in seconds).
Your data will **not** appear linear.
What type of function could model your data?
In a sentence or two, explain your choice.



Create Linear Data

Perhaps you hypothesized that a square root function models your data.
If so, you'll be pleased to know that a pendulum's period has the following form:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{where } g = \text{Earth's gravitational constant}$$

If we square both sides: $T^2 = \frac{4\pi^2}{g}L$

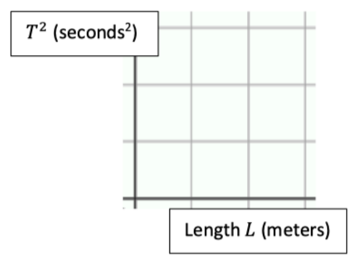
So, plotting T^2 versus L will result in a line with slope $\frac{4\pi^2}{g}$

Let's do that!

10.
Use your original table to create a new table.
Change the right-hand column, so that it now has the **squares** of the periods.

length L (meters)	**T^2 (seconds^2)**

11.
Plot your new data on a new grid.
Remember to place numbers, units, and labels on each axis.
Do your data now appear linear? Terrific!



12.
Determine the best-fit regression line that models your new data.
Do that this way:
Place your new data in your calculator. Also include the point $(0,0)$.
Calculate the line of best fit: DISTR, LINREG($ax + b$)
Record the resulting equation, carrying two decimals of accuracy for a and b.

Estimate Earth's Gravitational Constant g

13.
Since $T^2 = \frac{4\pi^2}{g}L$, your regression line's slope approximates $\frac{4\pi^2}{g}$
Use that insight to solve for g.

14.
You now have an experimental estimate for the gravitational constant g.
The actual value for gravity is: $g = 9.81$ meters/second^2
Calculate your experimental error as a percentage: $\frac{(your\ experimental\ value - actual\ value)}{actual\ value} \cdot 100 =$

15.
Your experimental value for g likely had error.
How could we improve this experiment to reduce that error?
In a well-written paragraph, describe at least three improvements.
If one suggestion is "measure more accurately", specify how you would accomplish that.

Measure Time

16.
Let's now adjust your pendulum so that it can serve as a clock, with period $T = 1$ second.
First, use $T = 2\pi\sqrt{\frac{L}{g}}$ to solve for L.
Then, replace $T = 1$ and $g = 9.81$ to calculate L accurate to three decimals (in meters).

17.
Now test that result:
Adjust your pendulum's string length to match your previous calculation.
Repeat step 3 one final time.
How accurately does your pendulum now measure time? A qualitative answer is fine.

18.
Suppose that we wish to build a lunar pendulum clock for astronauts to use on the moon.
The moon's gravitational constant is only one-sixth that of Earth's.
Calculate the length L that results in a period $T = 1$ second for that lunar pendulum clock.

19.
Galileo Galilei, Christiaan Huygens, and Robert Hooke all did pioneering work with pendulums.
For each, explain in a sentence of two how this exploration relates to his physics work.